CLASSICAL THEORY OF CHARGED MESON

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ABSTRACT. The equations of motion of the nucleon in interaction with the charged scalar meson field have been solved classically and the values of total scattering cross sections of charged mesons by nucleons have been calculated. The influence of radiation damping has also been investigated.

1. It is wellknown that the scattering cross sections of various processes become increasingly large with increasing energies if no account is taken of radiation reaction in the quantum mechanical calculations. However, it is not always possible to find exact solutions for scattering processes when the influence of radiation reaction is included in the mathematical formalism of quantum mechanics; it is often very difficult to estimate the degree of inaccuracy involved in the approximate solutions. Hence arises the necessity of approaching the problem from the classical theory in which the treatment of radiation reaction is much simpler. This scheme was originally introduced by Dirac (1938) in his 'classical theory of radiating electrons'. Bhabha (1939, 1941) and Harish Chandra (1946) extended the same method to the case of neutral meson field. It was not possible in Bhabha's theory to consider the charged mesons which are experimentally observed. The charge formalism was introduced by Fierz (1941) for the case of vector mesons, but he did not consider the dipole coupling. The complete treatment of electrically charged vector meson field was given by Le Couteur (1949). Recent experimental findings indicate, however, that the mesons involved in the nuclear interaction should be described by the pseudoscalar field. Now in a classical theory there should be no distinction between the scalar and pseudosealar interactions. Hence it is worth while to investigate the case of electrically charged scalar meson field.

2. In the charge formalism each field quantity is considered as a vector in a three dimensional space which we may call 'charge space', denoted by three unit vectors α, β, γ The electrical charge density – an observable quantity – is represented as the γ component of a vector (Le Couteur, 1949).

We shall mostly keep to the notation of Le Couteur's paper. We take the fundamental metric tensor $g_{\mu\nu}$ (the suffixes μ, ν, ρ, σ as usual run from 0 to 3) defined by $g_{00}=1$,

$$g_{11}=g_{22}=g_{33}=-1,$$

with all the other components vanishing. Then we may describe the charged scalar meson field in interaction with the nucleons by a potential $\mathbf{U}$ and field strengths $\mathbf{G}_\mu$ which statisfy the following equations :

$$\mathbf{G}'_\mu=\partial_\mu\mathbf{U}-4\pi\mathbf{S}_\mu=\mathbf{G}_\mu-4\pi\mathbf{S}_\mu, \quad \ldots \quad (1)$$

$$\partial^\mu\mathbf{G}'_\mu+\chi^2\mathbf{U}=4\pi\mathbf{S}, \quad \ldots \quad (2)$$

where $\chi=mc/\hbar$, m being the meson mass c and $\hbar$ having their usual meanings.

and
$$\partial_\mu=\frac{\partial}{\partial x^\mu}.$$

The source densities $\mathbf{S}$ and $\mathbf{S}_\mu$ are associated with the mesonic charge and dipole moment of the nucleons. The source densities arising from a single point particle may be assumed to be given by

$$\mathbf{S}=\int_{-\infty}^{\infty}\mathbf{s}(t)\delta(x-z(t))dt, \quad (3)$$

$$\mathbf{S}_\mu-\int^{\infty}\mathbf{s}_\mu(t)\delta(x-z(t))dt, \quad (4)$$

where $z_\mu(t)$ are the co-ordinates of the particle when its proper time is t (having the dimension of length) and $\mathbf{s}(t)$ and $\mathbf{s}_\mu(t)$ are continuous and differentiable functions of t.

From (1) and (2) we have

$$(\partial^\mu\partial_\mu+\chi^2)\mathbf{U}=4\pi(\mathbf{S}+\partial^\mu\mathbf{S}_\mu). \quad \ldots \quad (5)$$

Previously, in Bhabha's theory (Bhabha, 1940) it was not possible to consider a charged meson field classically because of the commutability of all quantities in the classical theory. This difficulty is removed by defining the charge current vector $J_\gamma{}^\mu$ as the 3rd. component of a vector product denoted by the symbol $\wedge$ -of two vectors in the charge space. Thus

$$4\pi\mathbf{J}^\mu=\left(\frac{c}{\hbar c}\right).\,\mathbf{G}'^\mu\wedge\mathbf{U}, \quad (6)$$

with the help of (1) and (2) we get

$$\partial^\mu\mathbf{J}_\mu=\left(\frac{c}{\hbar c}\right).\,\{\mathbf{S}\wedge\mathbf{U}-\mathbf{S}^\mu_\mu\wedge\mathbf{G}_\mu\}. \quad (7)$$

The energy momentum tensor is given by

$$4\pi T_{\mu\nu}=\mathbf{G}_\mu\mathbf{G}_\nu-\tfrac{1}{2}g_{\mu\nu}\{\mathbf{G}^\rho\mathbf{G}_\rho-\chi^2\mathbf{U}^2\}. \quad (8)$$

Hence we get after use of (1) and (2)

$$\partial^\nu T_{\mu\nu}=\mathbf{G}_\mu(\mathbf{S}+\partial^\rho\mathbf{S}_\rho)=\mathbf{G}_\mu\mathbf{S}-\partial^\rho\mathbf{G}_\mu\mathbf{S}_\rho+\partial^\rho(\mathbf{G}_\mu\mathbf{S}_\rho). \quad (9)$$

It is seen from (7) and (9) that in a source free region

$$\partial_\mu J_\gamma{}^\mu = 0 \text{ and } \partial^\mu T_{\mu\nu} = 0$$

The angular momentum of the meson field is given by

$$M_{\mu\nu,\lambda} = x_\mu T_{\nu\lambda} - x_\nu T_{\mu\lambda}. \quad \dots \quad (10)$$

Hence

$$\partial^\lambda M_{\mu\nu,\lambda} = x_\mu \partial^\lambda T_{\nu\lambda} - x_\nu \partial^\lambda T_{\mu\lambda}. \quad \dots \quad (11)$$

With the help of (9) we get

$$\partial^\lambda M_{\mu\nu,\lambda} = x_\mu \mathbf{S}\mathbf{G}_\nu - x_\mu \mathbf{S}_\lambda \partial^\lambda \mathbf{G}_\nu - \mathbf{S}_\mu \mathbf{G}_\nu$$
$$- x_\nu \mathbf{S}\mathbf{G}_\mu + x_\nu \mathbf{S}_\lambda \partial^\lambda \mathbf{G}_\mu + \mathbf{S}_\nu \mathbf{G}$$
$$+ \partial^\lambda \{ x_\mu \mathbf{G}_\nu \mathbf{S}_\lambda - x_\nu \mathbf{G}_\mu \mathbf{S}_\lambda \}. \quad \dots \quad (12)$$

Let us now determine the electric charge current density $J_\gamma{}^\mu$ and the average value of the energy momentum tensor for the special case of the plane wave as we shall require these afterwards for the evaluation of the scattering cross section. Following Le Couteur (1949) we have

$$U_\xi = A_\xi \sin (\omega_\mu x^\mu + \sigma_\xi), \ \xi = \alpha, \beta \text{ or } \gamma. \quad \dots \quad (13)$$

Then from equation (6)

$$J_\gamma{}^\mu = -\omega^\mu \left(\frac{c}{4\pi\hbar c} \right) A_\alpha A_\beta \sin (\sigma_\alpha - \sigma_\beta), \quad \dots \quad (14)$$

and with the help of equation (8) the average value of the energy momentum tensor is given by

$$T_{\mu\nu} = \frac{1}{8\pi} \omega_\mu \omega_\nu \mathbf{A}^2. \quad \dots \quad (15)$$

For a stream of positive or negative mesons we assume

$$A_\alpha = A_\beta, \ A_\gamma = 0 \text{ and } \sigma_\alpha = \sigma_\beta \mp \tfrac{1}{2}\pi, \quad \dots \quad (14a)$$

and in the case of neutral mesons

$$A_\alpha = A_\beta = 0.$$

3. The equations for the rates of change of the electric charge $\mathbf{Q}$, the energy momentum A_μ and the angular momentum $b_{\mu\nu}$ carried by the nucleon are derived from the method of inflow calculation given by Harish Chandra (1946). In these calculations we have to use modified mean fields defined by

$$\mathbf{U}'^{mean} = \mathbf{U}^{in} + \tfrac{1}{2}\mathbf{U}'^{rad},$$

and

$$\mathbf{G}_\mu{}'^{mean} = \mathbf{G}^{in} + \tfrac{1}{2}\mathbf{G}'^{rad}. \quad \dots \quad (16)$$

Hereafter we shall write $\mathbf{U}$ for $\mathbf{U}'^{mean}$ ane $\mathbf{G}_\mu$ for $\mathbf{G}'_\mu{}^{mean}$. Then from (7), (9) and (12)

$$\frac{d\mathbf{Q}}{dt} = \frac{e}{\hbar c} . \{ -\mathbf{s} \wedge \mathbf{U} + \mathbf{s}_\mu \wedge \mathbf{G}^\mu \} \quad \dots \quad (17)$$

$$\frac{dA_\mu}{dt} = -\mathbf{s}\mathbf{G}_\mu + \mathbf{s}_\rho\, \partial^\rho \mathbf{G}_\mu. \qquad \ldots \quad (18)$$

$$\frac{db_{\mu\nu}}{dt} = -z_\mu \mathbf{s}\mathbf{G}_\nu + z_\mu \mathbf{s}_\lambda \delta^\lambda \mathbf{G}_\nu + \mathbf{s}_\mu \mathbf{G}_\nu$$

$$+ z_\nu \mathbf{s}\mathbf{G}_\mu - z_\nu \mathbf{s}_\lambda \delta^\lambda \mathbf{G}_\mu - \mathbf{s}_\nu \mathbf{G}_\mu. \qquad \ldots \quad (19)$$

The spin angular momentum $B_{\mu\nu}$ is given by

$$\frac{dB_{\mu\nu}}{dt} = \frac{db_{\mu\nu}}{dt} - \frac{d}{dt}(z_\mu A_\nu - z_\nu A_\mu)$$

$$= (\mathbf{s}_\mu \mathbf{G}_\nu - \mathbf{s}_\nu \mathbf{G}_\mu) - (v_\mu A_\nu - v_\nu A_\mu), \qquad \ldots \quad (20)$$

where $$v_\mu = \frac{dz_\mu}{dt}.$$

We may split any vector into two parts (*i*) one orthogonal to velocity v_μ denoted by the sign $\sim$ overhead, and (*ii*) the other parallel to v_μ. Thus the dipole source function $\mathbf{s}_\mu$ is split as follows

$$\mathbf{s}_\mu = \tilde{\mathbf{s}}_\mu + \mathbf{S}^* v_\mu,$$

where $\tilde{\mathbf{s}}_\mu v^\mu = 0$ and $\mathbf{S}^* = \mathbf{s}_\mu v^\mu$.

It has been shown by Harish Chandra (1946) that with respect to the field producing properties the contribution of the part $\mathbf{S}^*$ can be included within that of the charge function $\mathbf{s}$. Hence without loss of generality we can take $\mathbf{s}_\mu = \tilde{\mathbf{s}}_\mu$.

Therefore

$$\mathbf{s}_\mu v^\mu = 0 \qquad \ldots \quad (21)$$

We now express the source functions $\mathbf{s}$ and $\mathbf{s}_\mu$ in terms of $\boldsymbol{\tau}$ the classical analogue of the quantum mechanical isotopic spin vector. We assume

$$\mathbf{s} = g\boldsymbol{\tau}, \qquad \ldots \quad (22)$$

$$\mathbf{s}_\mu = f s_\mu \boldsymbol{\tau}, \qquad \ldots \quad (23)$$

where g and f are coupling constants.

g has the dimenson of electric charge and f has the dimenson of electric dipole.

From (21) we see that $$s_\mu v^\mu = 0. \qquad \ldots \quad (21)$$

Thus in the rest system of co-ordinates $v_\mu = (1, 0, 0, 0)$, $s_0 = 0$ and the spatial parts are different from zero. In analogy with quantum mechanical results we assume that

$$\boldsymbol{\tau}^2 = 1 \quad \text{so that } \boldsymbol{\tau}\frac{d\boldsymbol{\tau}}{dt} = 0, \qquad \ldots \quad (24)$$

and $$s_\mu s^\mu = -1 \quad \text{whence } s_\mu \frac{ds^\mu}{dt} = 0. \qquad \ldots \quad (25)$$

Let $\epsilon_{\mu\nu\rho\sigma}$ be a tensor which is antisymmetric in each pair of indices and $\epsilon_{0123} = -1$.

Then
$$\tfrac{1}{2}\,\epsilon_{\mu\nu\rho\sigma} v^{\rho} s^{\sigma} .\, \epsilon^{\mu\nu\rho\sigma} v_{\rho} s_{\sigma} = 1. \tag{26}$$

Hence
$$\epsilon_{\mu\nu\rho\sigma} v^{\rho} s^{\sigma} . \frac{d}{dt}\left(\epsilon^{\mu\nu\rho\sigma} v_{\rho} s_{\sigma}\right) = 0. \tag{27}$$

Further
$$v^{\rho} s^{\sigma} \epsilon_{\mu\nu\rho\sigma} v^{\mu} = v^{\rho} s^{\sigma} \epsilon_{\mu\nu\rho\sigma} v^{\nu} = v^{\rho} s^{\sigma} \epsilon_{\mu\nu\rho\sigma} s^{\mu} = v^{\rho} s^{\sigma} \epsilon_{\mu\nu\rho\sigma} s^{\nu} = 0. \tag{28}$$

After use of (22) and (23), equation (17) is given by
$$\frac{d\mathbf{Q}}{dt} = -\frac{e}{\hbar c} .\, \boldsymbol{\tau} \wedge \{g\mathbf{U} - f s_{\mu} \mathbf{G}^{\mu}\}. \tag{29}$$

From this we get following Le Couteur
$$\frac{d\boldsymbol{\tau}}{dt} = \frac{2}{\hbar c}\, \boldsymbol{\tau} \wedge \{g\mathbf{U} - f s_{\mu}\mathbf{G}^{\mu} + \mathbf{q}\}. \tag{30}$$

Let us write
$$A_{\mu} = \tilde{A}_{\mu} + M^{*} v_{\mu}, \tag{31}$$

where $\tilde{A}_{\mu} v^{\mu} = 0$ and $M^{*} = A_{\mu} v^{\mu}$,
and
$$\mathbf{G}_{\mu} = \tilde{\mathbf{G}}_{\mu} + \mathbf{G}^{*} v_{\mu},$$

where
$$\tilde{\mathbf{G}}_{\mu} v^{\mu} = 0 \text{ and } \mathbf{G}^{*} = \mathbf{G}_{\mu} v^{\mu} = \frac{d\mathbf{U}}{dt} \tag{32}$$

Then contracting equation (20) with v^{ν} and using (21), (31) and (32), we get
$$\tilde{A}_{\mu} = \frac{dB_{\mu\nu}}{dt} v^{\nu} - \mathbf{s}_{\mu} \frac{d\mathbf{U}}{dt}, \tag{33}$$

$$\frac{dM^{*}}{dt} = v^{\mu} \frac{dA_{\mu}}{dt} + A_{\mu} \frac{dv^{\mu}}{dt}. \tag{34}$$

With the help of equations (33), (18) and (32), equation (34) may be written as
$$\frac{dM^{*}}{dt} = -g\boldsymbol{\tau} \frac{d\mathbf{U}}{dt} + f\boldsymbol{\tau} s_{\rho} \frac{d\mathbf{G}^{\rho}}{dt} + \frac{dB_{\mu\nu}}{dt} \frac{dv^{\mu}}{dt} v^{\nu} - f\boldsymbol{\tau} s_{\mu} \frac{d\mathbf{U}}{dt} \frac{dv^{\mu}}{dt}. \tag{35}$$

From (28) and (20) we see that
$$\epsilon^{\mu\nu\rho\sigma} v_{\rho} s_{\sigma} \frac{d\tilde{B}_{\mu\nu}}{dt} = 0.$$

On account of (27) and (28) we may take
$$\frac{d\tilde{B}_{\mu\nu}}{dt} = I \frac{d}{dt}\left((\epsilon_{\mu\nu\rho\sigma} v^{\rho} s^{\sigma}) - (s_{\mu} V_{\nu} - s_{\nu} V_{\mu}\right), \tag{36}$$

where I is a constant and V_μ is an arbitrary vector.

From (20) and (36) we have

$$I\epsilon_{\mu\nu\rho\sigma}\left(\frac{dv^\rho}{dt}s^\sigma + v^\rho\frac{ds^\sigma}{dt}\right) = s_\mu(f\tau\mathbf{G}_\nu + V_\nu) - s_\nu(f\tau\mathbf{G}_\mu + V_\mu)$$
$$-(v_\mu A_\nu - v_\nu A_\mu). \qquad (37)$$

Multiplying both sides by $\epsilon^{\mu\nu\alpha\beta}$ and using (Harish Chandra 1946)

$$\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\alpha\beta} = -2(\delta^\alpha_\rho\delta^\beta_\sigma - \delta^\alpha_\sigma\delta^\beta_\rho),$$

we get

$$I\left\{\left(\frac{ds^\alpha}{dt}v^\beta - \frac{ds^\beta}{dt}v^\alpha\right) + \left(s^\alpha\frac{dv^\beta}{dt} - s^\beta\frac{dv^\alpha}{dt}\right)\right\}$$
$$= s_\mu(f\tau\mathbf{G}_\nu + V_\nu)\epsilon^{\mu\nu\alpha\beta} - v_\mu A_\nu\epsilon^{\mu\nu\alpha\beta}. \qquad (38)$$

Omitting the parts proportional to v_μ or v_ν the equation (37) may be written as follows :

$$I\epsilon_{\mu\nu\rho\sigma}v^\rho\frac{ds^\sigma}{dt} = s_\mu\left(f\tau\widetilde{\mathbf{G}}_\nu + \widetilde{V}_\nu\right) - s_\nu\left(f\tau\widetilde{\mathbf{G}}_\mu + \widetilde{V}_\mu\right). \qquad (39)$$

Contracting equation (38) with $v_\beta\left(f\tau\mathbf{G}_\alpha + V_\alpha - f\tau v\frac{d\mathbf{U}}{dt}\right)$,

we obtain using (21), and $v_\mu\dfrac{dv^\mu}{dt} = 0$,

$$f\tau\frac{ds^\alpha}{dt}\left(\mathbf{G}_\alpha - v_\alpha\frac{d\mathbf{U}}{dt}\right) + \frac{ds^\alpha}{dt}V_\alpha - \frac{ds^\beta}{dt}v^\alpha v_\beta V_\alpha = 0 \qquad (40)$$

Also with the help of equation (28)

$$\frac{d}{dt}(\epsilon_{\mu\nu\rho\sigma}v^\rho s^\sigma)\frac{dv^\mu}{dt}v^\nu = -\epsilon_{\mu\nu\rho\sigma}v^\rho s^\sigma\frac{dv^\mu}{dt}\frac{dv^\nu}{dt} = 0 \qquad (41)$$

Hence from (21) and (30) we get

$$\frac{d\mathrm{B}_{\mu\nu}}{dt}\frac{dv^\mu}{dt}v^\nu = s_\mu V_\nu\frac{dv^\mu}{dt}v^\nu = \frac{ds_\mu}{dt}V_\nu v^\mu v^\nu. \qquad (42)$$

Therefore equation (35) may be written as

$$\frac{dM^*}{dt} = \frac{d}{dt}\left\{-g\tau\mathbf{U} + f\tau s_\mu\mathbf{G}^\mu\right\} + \frac{d\tau}{dt}\left\{g\mathbf{U} - fs_\mu\mathbf{G}^\mu\right\}$$
$$-f\tau\mathbf{G}_\mu\frac{ds^\mu}{dt} + f\tau\frac{ds_\mu}{dt}v^\mu\frac{d\mathbf{U}}{dt} + \frac{ds_\mu}{dt}V_\nu v^\mu v^\nu.$$

Using (40) and (30) we finally get

$$\frac{dM^*}{dt} = \frac{d}{dt}\left\{-g\tau\mathbf{U} + f\tau\mathbf{G}^\mu s_\mu\right\} - \frac{d\tau}{dt}\mathbf{q} + \frac{ds_\mu}{dt}V^\mu. \qquad (43)$$

Now $\mathbf{q}$ and V_μ may be so chosen that

$$\boldsymbol{\tau} \wedge \mathbf{q}, \ (s_\mu V_\nu - s_\nu V_\mu) \text{ and } \frac{d\boldsymbol{\tau}}{dt}\mathbf{q} - \frac{ds_\mu}{dt} V$$

are perfect differentials.

The simplest solution of course is $\mathbf{q}=0$, $V_\mu=0$. Another simple solution is

$$\mathbf{q} = -\tfrac{1}{2}L \frac{d^2\boldsymbol{\tau}}{dt^2}, \quad V_\mu = -\tfrac{1}{2}K \frac{d^2 s_\mu}{dt^2}. \qquad \ldots \quad (44)$$

With the help of (44) the intregration of (43) leads to

$$\mathrm{M}^* = \mathrm{m}^* - g\boldsymbol{\tau}\mathbf{U} + f\boldsymbol{\tau}\mathbf{G}^\mu s_\mu + \tfrac{1}{4}L\left(\frac{d\boldsymbol{\tau}}{dt}\right)^2 - \tfrac{1}{4}K \frac{ds^\mu}{dt}\frac{ds_\mu}{dt}. \qquad \ldots \quad (45)$$

Where m^* is the constant of integration.
Thus the equation of translational motion is solved completely.

4. We now evaluate the modified radiation fields $\mathbf{U}'^{\,rad}$ and $\mathbf{G}'^{\,rad}$ at points $z_\mu(t)$ on the worldline of the nucleon. Following the calculations given in Harish Chandra's paper (1946) we obtain

$$\begin{aligned}\mathbf{U}'^{\,rad}_{(z_\mu(t))} = & -2\frac{d\mathbf{s}}{dt} - 2\chi\int_{-\infty} \mathbf{s}\,\frac{J_1(\chi u)}{u}\,dt' \\ & -\tfrac{2}{3}\mathbf{s}_\sigma\frac{d^2v^\sigma}{dt} - 2\frac{d\mathbf{s}_\sigma}{dt}\frac{dv^\sigma}{dt} - 2\frac{d^2\mathbf{s}_\sigma}{dt^2}v^\sigma \\ & + 2\chi^2\int^t u^\sigma\mathbf{s}_\sigma\,\frac{J_2(\chi u)}{u^2}\,dt', \end{aligned} \qquad (46)$$

and $\mathbf{G}'^{\,rad}_{\mu(z_\mu(t))} =$

$$\begin{aligned} & -\tfrac{2}{3}\mathbf{s}\frac{d^2v_\mu}{dt^2} - 2\frac{d\mathbf{s}}{dt}\frac{dv_\mu}{dt} - 2\frac{d^2\mathbf{s}}{dt^2}v_\mu - \chi^2\mathbf{s}v_\mu \\ & \cdot 2\chi^2\int_{-\infty}^t u_\mu\mathbf{s}\,\frac{J_2(\chi u)}{u^2}\,dt' \\ & -\tfrac{2}{3}\mathbf{s}_\sigma v_\mu\frac{d^2v^\sigma}{dt^2} - \tfrac{8}{3}\frac{d\mathbf{s}_\sigma}{dt}\left(v_\mu\frac{d^2v^\sigma}{dt^2} + \frac{d^2v_\mu}{dt^2}v^\sigma\right) \\ & -4\frac{d^2\mathbf{s}_\sigma}{dt^2}\left(v_\mu\frac{dv^\sigma}{dt} + \frac{dv_\mu}{dt}v^\sigma\right) - \tfrac{8}{3}\frac{d^3\mathbf{s}_\sigma}{dt^3}v_\mu v^\sigma + \tfrac{2}{3}\frac{d^3\mathbf{s}_\mu}{dt^3} \\ & +\chi^2\Big(\frac{d\mathbf{s}_\mu}{dt} - \mathbf{s}_\sigma v^\sigma\frac{dv_\mu}{dt} - \mathbf{s}_\sigma v\frac{dv^\sigma}{dt} - 2\frac{d\mathbf{s}_\sigma}{dt}v_\mu v^\sigma \end{aligned}$$

$$+2\chi^3\int_{-\infty}^{t}\mathbf{s}_\mu\frac{J_2(\chi u)}{u^2}\,dt'-2\chi^3\int_{-\infty}^{t}u_\mu u^\sigma\mathbf{s}_\sigma\frac{J_3(\chi u)}{u^3}\,dt', \qquad \ldots (47)$$

where $u_\mu=x_\mu-z_\mu(t)$, x_μ being the field point and $u=|(u_\mu u^\mu)^{1/2}|$.

in the above calculations we have neglected terms of the second degree in $\frac{dv}{dt}$, $\frac{d^2v}{dt^2}$ and used the relation $\mathbf{s}_\sigma v^\sigma=0$.

In the rest system of co-ordinates $v_\mu=(1, 0, 0, 0)$, $u_1=u_2=u_3=0$ and $\mathbf{s}_0=0$ so that $u^\sigma\mathbf{s}_\sigma=0$,

and $u=t-t'$ whence, $\int_{-\infty}^{t}dt'=\int_{0}^{\infty}du$. ... (48)

Hence in this system of co-ordinates, equations (46) and (47) can be simplified as follows :

$$\mathbf{U}'^{\,rad}_{(z_\mu(t))}=-2g\frac{d\tau}{dt}-2g\chi\int_0^{\infty}\tau\frac{J_1(\chi u)}{u}\,du, \qquad (49)$$

and
$$\mathbf{G}'^{\,rad}_{(z_\mu(t))}=-2g\frac{d^2\tau}{dt^2}v_\mu-g\chi^2\tau v_\mu+2g\chi^2\int_0\tau u_\mu\frac{J_2(\chi u)}{u^2}\,du$$
$$+\frac{2}{3}\frac{d^3\mathbf{s}_\mu}{dt^3}+\chi^2\frac{d\mathbf{s}_\mu}{dt}+2\chi\int_0\mathbf{s}_\mu\frac{J_2(\chi u)}{u^2} \qquad (50)$$

Further equation (39) may be written in view of equation (44)

as
$$\frac{dM}{dt}\quad M, f\tau\mathbf{H}-K\frac{d^2M}{dt^2}\Big], \qquad (51)$$

where M is a three dimensional vector in ordinary space with components s_1, s_2, s_3 ; $M_kM_k=1$, and $\tilde{\mathbf{H}}$ is a vector in ordinary space with components $\tilde{\mathbf{G}}_1$, $\tilde{\mathbf{G}}_2$ and $\tilde{\mathbf{G}}_3$. [A, B] denotes vector product of vectors A and B in ordinary three dimensional space and (A B) denotes their scalar product. Similarly equation (30) becomes

$$\frac{d\tau}{dt}=\frac{2\tau}{\hbar c}\Lambda\Big\{g\mathbf{U}-f(M\tilde{\mathbf{H}})-\tfrac{1}{2}L\frac{d^2\tau}{dt^2} \qquad \ldots (52)$$

With the help of (48) and (50) we have

$$\tilde{\mathbf{H}}=\tilde{\mathbf{H}}^{in}+\tfrac{1}{3}f\left(\tau\frac{d^3M}{dt^3}+3\frac{d\tau}{dt}\frac{d^2M}{dt^2}+3\frac{d^2\tau}{dt^2}\frac{dM}{dt}+\frac{d^3\tau}{dt^3}M\right)$$
$$+\tfrac{1}{2}\chi^2f\left(\tau\frac{dM}{dt}+\frac{d\tau}{dt}M\right)+\chi^2f\int_0^{\infty}\tau M\frac{J_2(\chi u)}{u^2}\,du.$$

Hence equations (51) and (52) become

$$I\frac{dM}{dt} \quad \Big| M, f\tau\widetilde{\mathbf{H}}^{in} - \tfrac{1}{2}K\frac{d^2M}{dt^2}$$

$$\cdot f^2\left[M, \tfrac{1}{3}\frac{d^3M}{dt^3} + \tau\frac{d^2\tau}{dt^2}\quad\frac{dM}{dt} + \frac{\chi^2}{2}\frac{dM}{dt} + \chi^2\tau\int_0^{\alpha}\tau M\frac{J_2(\chi u)}{u^2}du \qquad (51)$$

and

$$\frac{d\tau}{dt} = \frac{2\tau}{\hbar c}\wedge \quad g\mathbf{U}^{in} - f(\mathbf{MH}\quad)\Big\} - \frac{L\tau}{\hbar c}\wedge\frac{d^2\tau}{dt^2}$$

$$-\frac{2\tau}{\hbar c}\wedge g^2\Big\{\frac{d\tau}{dt} + \chi\int_0 \tau\frac{J_1(\chi u)}{u}du$$

$$-\frac{2\tau}{\hbar c}\wedge f^2\Big\}\quad M\frac{d^2M}{dt^2}\Big)\frac{d\tau}{dt} + \frac{M^2}{3}\frac{d^3\tau}{dt^3} + \frac{\chi^2M^2}{2}\frac{d\tau}{dt} + \chi^2M_t\int^{\alpha}\tau M_t\frac{J_2(\chi u)}{u^2}du\Big\{ \qquad \ldots (52)$$

5. **We** now try to solve the above equations assuming the incident field to be **weak and** simply periodic with frequency ω. Equation (51) agrees with Le Couteur's equation (61) (Le Couteur, 1949), the solution of which is given by Bhabha in his neutral theory. We shall neglect the periodic oscillation of velocity.

To solve equation (52) we assume wlth Le Couteur that

$$\tau = a\gamma + \varepsilon_1\alpha\sin(\omega t + \sigma_1) + \varepsilon_2\beta\sin(\omega t + \sigma_2), \qquad (53)$$

where $a \approx \pm 1$ depending upon whether the particle is initially a neutron or a proton, further ε_1 and ε_2 are quantities of smaller magnitude. To evaluate the radiation damping terms we notice that the terms proportional to f^2 are the same as in Le Couteur's paper except for a factor $\frac{1}{2}$. Hence we need evaluate only the terms proportioual to g^2. Ws shall carry our calculation for the case $\omega > \chi$ as needed for the evaluation of the scattering cross section. Thus we get the radiation damping term

$$-Pa\gamma + R\varepsilon_1\alpha\cos(\omega t + \sigma_1) + R\varepsilon_2\beta\cos(\omega t + \sigma_2), \qquad \ldots (54)$$

where $\quad P = 2g^2\chi + \tfrac{2}{3}f^2M^2\chi^3,$

and $\quad R = -2g^2(\omega^2 - \chi^2)^{1/2} + \tfrac{2}{3}f^2M^2(\omega^2 - \chi^2)^{3/2}. \qquad \ldots (55)$

Substituting (54) in (52) we obtain two equations by equating the coefficients of α and β on both sides. We write

$$\Big\{g\mathbf{U}^{in} - f\Big(M\widetilde{\mathbf{H}}^{in}\Big)\Big\}\alpha = \delta_1\sin\omega t + \delta_2\cos\omega t,$$

$$\Big\{g\mathbf{U}^{in} - f\Big(M\widetilde{\mathbf{H}}^{in}\Big)\Big\}\beta = \delta_1\sin\Big(\omega t \pm \frac{\pi}{2}\Big) + \delta_2\cos\Big(\omega t \pm \frac{\pi}{2}\Big). \qquad \ldots (56)$$

Comparing with (14 a) we associate the $\pm$ signs with the positive or the negative meson. Further we take

$$\sigma_2=\sigma_1\pm\pi/2 \text{ and } \epsilon_1=\epsilon_2=\epsilon, \qquad \ldots \quad (57)$$

where σ_1 and ϵ are determined by the following relations :

$$\frac{\cos\sigma_1}{\delta_2 R+\delta_1\left(\pm\frac{\omega\hbar c}{a}+L\omega^2+P\right)}=\frac{\sin\sigma_1}{\delta_2\left(\pm\frac{\omega\hbar c}{a}+L\omega^2+P\right)-\delta_1 R} \qquad \ldots \quad (58)$$

and

$$\epsilon^2=4(\delta_1^2+\delta_2^2)\left\{\left(\frac{\omega\hbar c}{a}+L\omega^2+P\right)^2+R^2\right\}^{-1}$$

$$=\frac{4}{\hbar^2c^2}\cdot\frac{\delta_1^2+\delta_2^2}{\omega^2}\left[\left\{1\pm\frac{a}{\hbar c}\left(L\omega+\frac{P}{\omega}\right)\right\}^2+\frac{R^2}{\omega^2\hbar^2c^2}\right]^{-1} \qquad \ldots \quad (59)$$

6. The contribution of $\frac{dM}{dt}$ to the scattering cross section is of secondary importance and we need consider the contribution of $\frac{d\tau}{dt}$ only. The variations of the charge function and the dipole function are respectively given by $g\frac{d\tau}{dt}$ and $fM\frac{d\tau}{dt}$. The retraded potentials induced are respectively

$$\frac{\epsilon g}{r}\left[\alpha\sin\left\{\omega t-\sqrt{\omega^2-\chi^2}.r+\sigma_1\right\}+\beta\sin\left\{\omega t-\sqrt{\omega^2-\chi^2}.r+\sigma_2\right\}\right], \qquad \ldots \quad (60)$$

and

$$\epsilon f\sqrt{\omega^2-\chi^2}\,\frac{(rM)}{r^2}\left[\alpha\cos\left\{\omega t-\sqrt{\omega^2-\chi^2}.\,r+\sigma_1\right\}\right.$$

$$\left.+\beta\cos\left\{\omega t-\sqrt{\omega^2-\chi^2}.\,r+\sigma_2\right\}\right]. \qquad \ldots \quad (61)$$

The average of radial current density over all directions of orientation of M according to (14) and (14a) is given by

$$\left(\frac{e}{4\pi\hbar c}\right)\sqrt{\omega^2-\chi^2}\,\frac{1}{r^2}\left\{g^2+f^2M^2(\omega^2-\chi^2)\,\overline{\cos^2\phi}\right\}\epsilon^2 \qquad \ldots \quad (62)$$

where ϕ is the angle between r and M and the bar denotes an average over all possible directions of r with respect to M

Let the incident meson wave be given by

$$\mathbf{U}=A\left[\alpha \sin\left\{\omega x^0-\sqrt{\omega^2-\chi^2}\,.\,x^1\right\}+\beta \sin\left\{\omega x^0-\sqrt{\omega^2-\chi^2}\,.\,x^1\pm\frac{\pi}{2}\right\}\right], \quad \ldots \quad (63)$$

the corresponding current density being given by

$$\pm\left(\frac{e}{4\pi\hbar c}\right).\sqrt{\omega^2-\chi^2}\,.\,A^2 \quad \ldots \quad (64)$$

Comparing equation (63) with (56) we get the values of δ_1 and δ_2 and substitute these in (59) to calculate ϵ^2. Thus from (62) and (64) the total cross section is given by

$$4\pi\left\{g^2+f^2M^2(\omega^2-\chi^2)\overline{\cos^2\phi}\right\}.\frac{\epsilon^2}{A^2}$$

$$\frac{16\pi}{\omega^2\hbar^2c^2}\,\frac{\left\{g^2+f^2M^2(\omega^2-\chi^2)\overline{\cos^2\phi}\right\}\left\{g^2+f^2M^2(\omega^2-\chi^2)\overline{\cos^2\theta}\right\}}{\left\{1\pm\frac{a}{\hbar c}\left(L\omega+\frac{P}{\omega}\right)\right\}^2+\frac{R^2}{\omega^2\hbar^2c^2}}, \quad \ldots \quad (65)$$

where θ is the angle between M and $\mathbf{H}^{in}$.
In the quantum theory of nucleon spin $\frac{1}{2}$ we have

$$\overline{\cos^2\phi}=\overline{\cos^2\theta}=1 \text{ and } M^2=3. \quad \ldots \quad (66)$$

7. To compare with experimental results we put $L=0$ as has been done by Le Couteur (1949). As the values of the coupling constants in the pseudoscalar field are arbitrary to the extent of the cut-off distance, we assume, for our purpose here, their values to be the same as determined for the Möller-Rosenfeld mixed field from the deuteron binding energies; $g^2/\hbar c=.024$ and $f^2\chi^2/\hbar c=.096$. The theoretical results are shown in curves. I and II in figure 1. curve I ($\pm a=-1$) gives the scattering of (negative/positive) mesons by (neutrons/protons) and curve II ($\pm a=1$) gives that of (negative/positive) mesons by (protons/neutrons). curve I shows that the maximum value of the scattering cross section is reached when the incident meson energy is 210 Mev; in case of curve II the corresponding meson energy is 270 Mev. The experimental results, though not very reliable, indicate that the maximum value of the cross section of the scattering of positive meson by proton is reached for 180 Mev energy of the incident meson. It may be mentioned here that the maximum values as obtained by Le Couteur (1949) for the above cross sections in the vector field with the same coupling constants occur round about 140 Mev meson energy. So it appears that in the

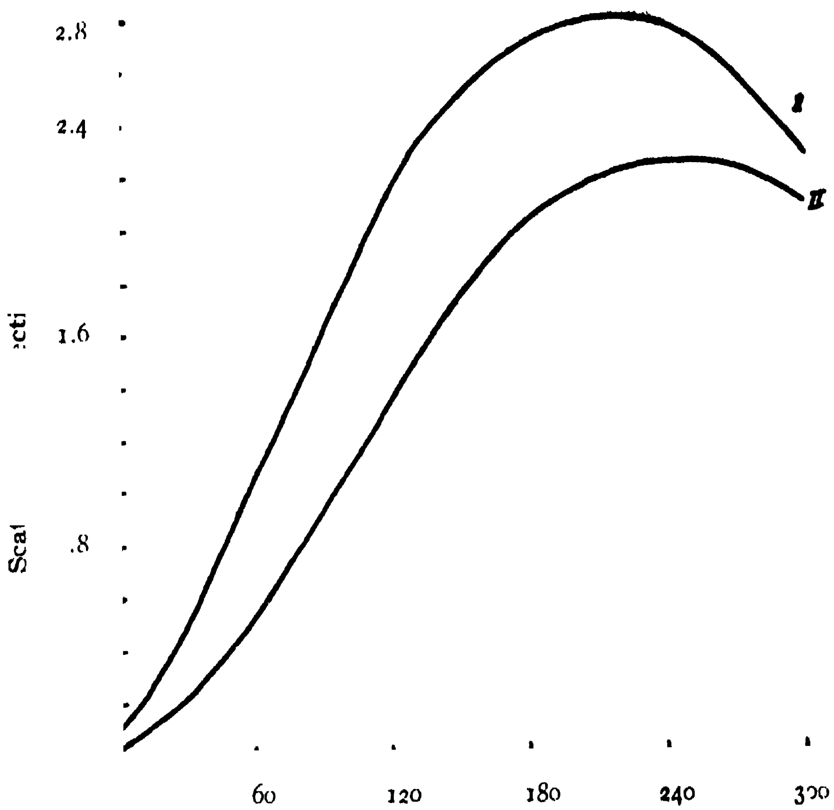

FIG. 1

Meson energy in Mev

pseudoscalar field the maximum value of the cross section is reached at meson energies higher than that in the vector field.

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REFERENCES

Bhabha, 1939, *Proc. Roy. Soc A.*, **172**, 384.

,, 1940, *Proc. Ind. Acad. A.*, **11**, 347.

,, 1941, *Proc. Roy. Soc A.*, **178**, 314.

Dirac, 1938, *Proc. Roy. Soc. A.*, **167**, 148.

Fierz, 1941, *Helv. Phys. Acta.*, **14**, 257.

Harish Chandra, 1946, *Proc. Roy. Soc. A.*, **186**, 269.

Le Couteur, 1949, *Proc Cambridge. Phil. Soc.*, **46**, 429.